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# Accelerated Depreciation, Default Risk and Investment Decisions 

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#### Abstract

In this article we focus on a representative firm that can decide when to invest under default risk. On the one hand, this firm can benefit from generous tax depreciation allowances, on the other hand it faces a default risk. Our aim is to study the effects of tax depreciation allowances in a risky environment. As will be shown in our numerical analysis, generous tax depreciation allowances lead to a decrease in a firm's leverage and, in most cases, cause a reduction in default risk. This result has a strong policy implication, in that it shows that an investment stimulus pack is expected neither to increase the default risk nor to cause financial instability.


JEL-codes: H200.
Keywords: capital structure, contingent claims, corporate taxation and hybrid securities.

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## 1 Introduction

The last decade has been characterized by the well-known Great Recession which caused a substantial increase in financial instability. ${ }^{1}$ In 2008, the G20 committed to a broad range of policy reforms that addressed the major fault lines that caused the crisis. At the same time, many Governments introduced fiscal stimulus packs aimed mainly at stimulating investment during the recession.

Over the last two decades, a part of the relevant literature has been studying the effects of taxation on firms' choices, thereby analyzing the interactions between financial and start-up decisions in a dynamic context (with or without complete information). ${ }^{2}$ Other articles have focused on the neutrality properties of taxation in a real-option setting, ${ }^{3}$ as well as the non-linear tax effects on investment (e.g., Gries et al., 2012). Despite the growing interest in these topics, only Mauer and Ott (1995) have analyzed the effects of tax depreciation allowances on investment decisions. However, they only focused on an equity-financed investment, thereby disregarding default risk. ${ }^{4}$

In this article we aim to study the effects of tax depreciations allowances on both real and financial decisions. In particular, we will focus on a representative firm that can decide when to invest and how much to borrow, under default risk. This joint analysis allows us to understand the effects of accelerated depreciation allowances in a risky environment. Given the capital structure of a firm, it is straightforward to show that accelerated depreciation stimulates investment. This means that, due to the tax benefit, a firm decides to invest with a lower initial EBIT. Coeteris paribus however,

[^0]a lower EBIT implies a higher probability of default. If this is true, accelerated depreciation may cause an increase in bankruptcies and thus lead to higher systemic risk. Our model shows that, using realistic parameter values, accelerated depreciation delays the event of default. This result has an interesting policy implication: generous tax depreciation allowances are expected to stimulate investment without increasing systemic risk.

The structure of this article is as follows. In Section 2, we develop a continuous-time model describing a representative firm that can decide both investment timing and the leverage ratio. Since the relationship between investment and financial choices is non-linear, we cannot find a closed-form solution. For this reason, Section 3 provides a numerical analysis. Section 4 summarizes our findings and discusses their policy implications.

## 2 The model

In this section we introduce an EBIT-based model in the spirit of Goldstein et al. (2001).

Let us assume that a company starts to earn an EBIT, denoted by $\Pi_{t}$, once a depreciable investment cost $I$ has been paid. Moreover, we introduce the following:

Assumption 1 A firm's EBIT evolves according to a geometric Brownian motion, where $\sigma$ is the instantaneous standard deviation and $d z_{t}$ is the increment of a standard Wiener process. Moreover, at any time $t$ there is a probability $\lambda d t$ that the existing project dies during the short internal dt.

Given Assumption 1, a firm's EBIT evolves as follows:

$$
\begin{equation*}
\frac{d \Pi_{t}}{\Pi_{t}}=\alpha d t+\sigma d z_{t}-d q_{t}, \text { with } \Pi_{0}>0 \tag{1}
\end{equation*}
$$

where $d q_{t}$ is the increment of a Poisson process with arrival rate $\lambda$. Under eq. (1), therefore, the expected lifetime of an investment project is finite, although uncertain. As shown by Dixit and Pindyck (1994, p. 270), the expected time until the Poisson jump occurs is $E(T)=1 / \lambda$. For simplicity, below we will omit the time variable.

Capital markets and debt Let us assume that risk is fully diversifiable, credit markets are perfectly competitive and information is symmetric.

Debt causes both costs and benefits. On the one hand, debt finance may lead to default. On the other, the tax deductibility of interest payments ensures levered companies a saving (Modigliani and Miller, 1963; Leland, 1994).

When a company does not meet its debt obligation, default takes place. In this case, shareholders are expropriated by the lender. Denoting $C$ as the coupon paid to the lender, we introduce the following:

Assumption 2 At time 0, the company issues a consol bond and pays a coupon $C$, which is not renegotiable.

According to Assumption 2 a company sets a coupon. Given $C$ it is straightforward to find the mark-to-market value of debt. For simplicity, we also assume that debt cannot be renegotiated: this means that we apply a static model, where a company's financial policy cannot be reviewed later. ${ }^{5}$

Assumption 3 If $\Pi$ drops to a threshold value, default occurs.
Assumption 4 The cost of default is $v C$ with $v>0$.
Assumptions 3 and 4 introduce the risk and the cost of default, respectively. Given (1), it is assumed that, if a company's EBIT falls to a given threshold value, the company cannot meet its obligation and is fully expropriated by the lender (Assumption 3). In the event of default, the lender faces a sunk cost, which is proportional to the coupon paid (Assumption 4). ${ }^{6}$

Taxation Let us next focus on the tax system. Given the corporate tax rate $\tau$, we introduce the following:

Assumption 5 Interest payments are fully deductible.
Assumption 6 A straight-line tax depreciation allowance, equal to $\lambda_{F}$ times the investment cost I, is granted throughout the investment's lifetime.

[^1]Assumption 7 Before default, the lender's tax rate is nil. After default, the lender becomes shareholder and the relevant tax rate is $\tau$.

Assumption 8 The tax system is fully symmetric.
Assumption 5 introduces the tax benefit of interest deductibility. ${ }^{7}$ Assumption 6 describes a simple straight-line fiscal depreciation allowance which is granted throughout the investment's lifetime. When the project dies, this deduction vanishes. To analyze the expected present value of such depreciation allowance we must consider that the expected lifetime of an investment project is $E(T)=1 / \lambda$. We can therefore say that, at any time $t$, a company can deduct $\lambda_{F} I$, thereby enjoying a benefit equal to $\tau \lambda_{F} I$. If the equality $\lambda_{F}=\lambda$ holds, depreciation allowances are such that all the investment costs are expected to be amortized during a firm's lifetime. If $\lambda_{F}>\lambda$, fiscal depreciation allowance is more generous than economic depreciation (in this case more than $100 \%$ of $I$ is amortized). The converse is true if $\lambda_{F}<\lambda$.

Assumption 7 accounts for the fact that, in most cases, the tax burden on capital income is relatively low or even close to zero. In this model, the lender's tax rate is equal to zero for simplicity. Note however that, after default, the lender becomes shareholder and is thus subject to corporate taxation. For simplicity, according to Assumption 8, the treatment of profit and loss is symmetric. ${ }^{8}$

Given these assumptions we can write a company's after-tax payoff as:

$$
\begin{equation*}
\Pi^{N}(\Pi ; C)=(1-\tau)(\Pi-C)+\tau \lambda_{F} I . \tag{2}
\end{equation*}
$$

Let us next calculate the value of equity and debt, respectively. Using dynamic programming, we can write the value of equity as the sum between the after-tax payoff received in the interval $d t$, i.e., $\Pi^{N}(\Pi ; C) d t$, and the

[^2]value function after the time interval $d t$ has passed: ${ }^{9}$

$E(\Pi ; C)=\left\{\begin{array}{lc}0 & \text { after default }, \\ \Pi^{N}(\Pi ; C) d t+(1-\lambda d t) e^{-r d t} \mathbb{E}[E(\Pi+d \Pi ; C)] & \text { before default },\end{array}\right.$
where $\mathbb{E}[\cdot]$ is the expectation operator. As shown in (3), at any period $d t$, there is a probability $\lambda d t$ that the project dies and that the value of equity goes to zero. Following the same procedure, we can calculate the mark-tomarket value of debt:
$D(\Pi ; C)= \begin{cases}\Pi^{N}(\Pi ; 0) d t+(1-\lambda d t) e^{-r d t} \mathbb{E}[D(\Pi+d \Pi ; C)] & \text { after default, } \\ C d t+(1-\lambda d t) e^{-r d t} \mathbb{E}[D(\Pi+d \Pi ; C)] & \text { before default. }\end{cases}$
As can be seen, the value of debt is contingent on the event of sudden death of capital. Since there is a probability $\lambda d t$ that a company's profit goes to zero, in this case, the lender's claim becomes worthless. This means that, given Assumption 1, debt financing is in line with the expected lifetime of investment. ${ }^{10}$ Note that, by assuming this fact, we depart from most of the existing theoretical literature, which usually assumes that investment is financed only with default-free short-term debt, irrespective of a project's expected lifetime and riskiness (e.g., Devereux and Griffith, 1999).

## 3 Debt finance

In this section we introduce two different kinds of debt: secured and unsecured. ${ }^{11}$ Using an EBIT-based model, we can say that:

Definition 1 Under secured debt finance, default occurs when $\Pi$ falls to an exogenously given threshold point $\widetilde{\Pi}^{s}$.

[^3]Definition 2 Under unsecured debt finance, the threshold point, denoted as $\widetilde{\Pi}^{u}$, is chosen optimally by shareholders.

According to Definition 1, default may be triggered when a company's EBIT falls to the exogenously given threshold point $\widetilde{\Pi}^{s}$. This definition refers to secured debt, where default takes place when a company's asset value falls to the debt's value. ${ }^{12}$

Under Definition 2, when a company's net cash flow is negative, shareholders can decide whether to inject further equity capital in order to meet their company's debt obligations or to default. As long as they issue new capital and pay the coupon, they can exploit a future recovery in profitability. Under unsecured debt finance, therefore, shareholders behave as if they owned a put option, the exercise of which leads to default.

Let us next calculate the value of equity. Using (3), applying Itô's Lemma and rearranging gives the following non-arbitrage condition:
$(r+\lambda) E^{j}(\Pi ; C)=\Pi^{N}(\Pi ; C)+\alpha \Pi E_{\Pi}(\Pi ; C)+\frac{\sigma^{2}}{2} \Pi^{2} E_{\Pi \Pi}(\Pi ; C)$ for $\Pi>\widetilde{\Pi}^{j}$,
where $E_{\Pi}(\Pi ; C) \equiv \frac{\partial E(\Pi ; C)}{\partial \Pi}$ and $E_{\Pi \Pi}(\Pi ; C) \equiv \frac{\partial E^{2}(\Pi ; C)}{\partial \Pi^{2}}$. As can be seen, in equation (5) the relevant discount rate is $r+\lambda$ instead of $r$. As explained by Dixit and Pindyck (1994, p. 200), to deal with a stochastic decay of capital "we can regard the project as infinite-live, but augment the rate at which future profits are discounted by adding the Poisson death parameter". Solving (5) we obtain (see Appendix A):

$$
E^{j}(\Pi ; C)= \begin{cases}0 & \Pi<\widetilde{\Pi}^{j}  \tag{6}\\ \Psi(\Pi ; C)-\Psi\left(\widetilde{\Pi}^{j} ; C\right)\left(\frac{\Pi}{\widetilde{\Pi}^{j}}\right)^{\beta_{2}(\lambda)} & \Pi>\widetilde{\Pi}^{j}\end{cases}
$$

with $j=s, u$ and $\Psi(\Pi ; C) \equiv(1-\tau)\left(\frac{\Pi}{\delta+\lambda}-\frac{C}{r+\lambda}\right)+\tau \Omega I$, where $\Omega \equiv \frac{\lambda_{F}}{r+\lambda}$ is the present value of fiscal depreciation allowances and $\beta_{2}(\lambda)=\frac{1}{2}-\frac{\alpha}{\sigma^{2}}-$ $\sqrt{\left(\frac{\alpha}{\sigma^{2}}-\frac{1}{2}\right)^{2}+\frac{2(r+\lambda)}{\sigma^{2}}}<0$. The term $\Psi(\Pi ; C)$ accounts for a scenario where no change occurs, apart from the future death of the project. As can be seen, the relevant discount rate for $\Pi$ is $\delta+\lambda$, where $\delta$ is the so-called convenience (or dividend) yield. ${ }^{13}$ The latter term measures the contingent value

[^4]of default. In particular, $\left(\Pi / \widetilde{\Pi}^{j}\right)^{\beta_{2}(\lambda)}$ measures the present value of 1 Euro contingent on the default event, and $\Psi\left(\widetilde{\Pi}^{j} ; C\right)$ is the expected present value of the profit lost by shareholders after expropriation. As can be seen, an increase in $\Pi$ raises current inflows and reduces the probability of default. This means that the value of equity is positively affected by the current value of $\Pi$.

Given these results, we can now calculate the default threshold points under secured and unsecured debt, respectively.

Secured debt According to Definition 1, full debt protection means that the default threshold point $\widetilde{\Pi}^{s}$ must be such that we have $\Pi^{N}\left(\widetilde{\Pi}^{s} ; C\right)=$ 0 Using (2) gives:

$$
\begin{equation*}
\widetilde{\Pi}^{s}=C-\frac{\tau}{1-\tau} \lambda_{F} I \tag{7}
\end{equation*}
$$

As can be seen, the higher the tax depreciation rate $\lambda_{F}$, the lower the threshold level $\widetilde{\Pi}^{s}$. Of course, this leads to a delay of default.

Unsecured debt To calculate the threshold value under unsecured debt we follow Leland (1994). Accordingly, $\widetilde{\Pi}^{u}$ is obtained by maximizing the value of equity, i.e.,

$$
\begin{equation*}
\max _{\widetilde{\Pi}^{u}} E(\Pi, C) . \tag{8}
\end{equation*}
$$

Solving problem (8) (see Appendix B), we obtain:

$$
\begin{equation*}
\widetilde{\Pi}^{u}=\frac{\beta_{2}(\lambda)}{\beta_{2}(\lambda)-1} \frac{\delta+\lambda}{r+\lambda}\left(C-\frac{\tau}{1-\tau} \lambda_{F} I\right)<\widetilde{\Pi}^{s} . \tag{9}
\end{equation*}
$$

Like the secured-debt case, the higher the tax depreciation rate $\lambda_{F}$, the lower the threshold level $\widetilde{\Pi}^{s}$ is. Moreover, the inequality $\widetilde{\Pi}^{u}<\widetilde{\Pi}^{s}$ holds. This result can be explained as follows: under unsecured debt finance, a company can inject equity in order to meet its debt obligations. This means that, unlike the secured-debt finance case, a company can postpone default. This means that contingent cost of default is higher under unsecured debt, i.e., $\left(\Pi / \widetilde{\Pi}^{u}\right)^{\beta_{2}(\lambda)}>\left(\Pi / \widetilde{\Pi}^{s}\right)^{\beta_{2}(\lambda)} \cdot{ }^{14}$ Moreover, Appendix C shows that $0>\frac{\partial \widetilde{\Pi}^{u}}{\partial \lambda_{F}}>$

[^5]$\frac{\partial \widetilde{\Pi}^{s}}{\partial \lambda_{F}}$. This means that the effect of $\lambda_{F}$ on the contingent evaluation of future events crucially depends on the characteristics of debt.

Following the same procedure, we can now calculate the value of debt. As shown in Appendix D, we obtain:

$$
D^{j}(\Pi ; C)= \begin{cases}\frac{(1-\tau) \Pi}{\delta+\lambda}+\tau \Omega I & \Pi<\widetilde{\Pi}^{j}  \tag{10}\\ \frac{C}{r+\lambda}+\left[\frac{(1-\tau) \widetilde{\Pi}^{j}}{\delta+\lambda}+\tau \Omega I-\frac{C}{r+\lambda}-v C\right]\left(\frac{\Pi}{\widetilde{\Pi}^{j}}\right)^{\beta_{2}(\lambda)} & \Pi>\widetilde{\Pi}^{j}\end{cases}
$$

Function (10) shows that the value of debt depends not only on $C$ but also on the current value of $\Pi$. If $\Pi>\widetilde{\Pi}^{j}$, the value of debt consists of two terms: a perpetual rent and a term that is non-linear in $\Pi$. In this case, an increase in $\Pi$ reduces the probability of default and therefore raises the value of debt. As can be seen, the lender's relevant discount rate is $r+\lambda$ : this means that the expected lifetime of debt (until default) is in line with the expected lifetime of investment. The second term measures the contingent value of the net cost of default. After default (i.e., when $\Pi$ has reached $\Pi^{j}$ ), the lender becomes shareholder and the value of his/her company is equal to $\left[\frac{(1-\tau) \widetilde{\Pi}^{j}}{\delta+\lambda}+\tau \Omega I\right]$. In this case, the firm is fully equity-financed and its value is positively affected by $\Pi$.

Using (6) and (10) we obtain the value of the levered firm:

$$
\begin{equation*}
V^{j}(\Pi ; C)=E^{j}(\Pi ; C)+D^{j}(\Pi ; C) \tag{11}
\end{equation*}
$$

As can be seen, $V^{j}(\Pi ; C)$ depends on both the tax rate and the default cost. Of course, it also accounts for the fact that, in the event of default, the tax benefit of interest deductibility is lost.

## 4 The option to invest

Let us next focus on investment timing. We let a representative firm decide when to invest a given amount of resources $I$. According to Dixit and Pindyck (1994), this means that our representative firm owns an investment option, with the following functional form:

$$
O(\Pi)=A_{1} \Pi^{\beta_{1}},
$$

where $A_{1}$ is an unknown to be determined and $\beta_{1}=\frac{1}{2}-\frac{\alpha}{\sigma^{2}}+\sqrt{\left(\frac{\alpha}{\sigma^{2}}-\frac{1}{2}\right)^{2}+\frac{2 r}{\sigma^{2}}}>$ $1 .{ }^{15}$ In order to find the optimal investment timing, we also need to calculate a business' Net Present Value (NPV). Using (11), the NPV will be equal to:

$$
\begin{aligned}
N P V^{j}(\Pi ; C) & =V^{j}(\Pi ; C)-I=V_{U}(\Pi ; C)+B^{j}(C)-I \\
& =\left[\frac{(1-\tau) \Pi}{r+\lambda}-(1-\Omega \tau) I\right]+\left\{\tau-[\tau+(r+\lambda) v]\left(\frac{\Pi}{\widetilde{\Pi}^{j}}\right)^{\beta_{2}(\lambda)}\right\} \frac{C}{r+\lambda}
\end{aligned}
$$

with $j=s, u$. Following Panteghini (2007b), a firm's objective function will then be equal to $\left(\frac{\Pi}{\Pi^{*}}\right)^{\beta_{1}}$ times the NPV, i.e.,

$$
\begin{equation*}
W^{j}=\left(\frac{\Pi}{\Pi^{*}}\right)^{\beta_{1}}\left\langle\left[\frac{(1-\tau) \Pi^{*}}{r+\lambda}-(1-\Omega \tau) I\right]+\left\{\tau-[\tau+(r+\lambda) v]\left(\frac{\Pi^{*}}{\widetilde{\Pi}^{j}}\right)^{\beta_{2}(\lambda)}\right\} \frac{C}{r+\lambda}\right\rangle \tag{12}
\end{equation*}
$$

with $j=s, u$. Term $\left(\frac{\Pi}{\Pi^{*}}\right)^{\beta_{1}}$ measures the contingent value of 1 Euro, when $\Pi<\Pi^{*}$, which will be invested whenever $\Pi=\Pi^{*}$. Maximizing (12) with respect to $\Pi^{*}$ gives the following first order condition

$$
\begin{align*}
\frac{\partial W^{j}}{\partial \Pi^{*}}= & -\beta_{1} \Pi^{*^{-1}}\left(\frac{\Pi}{\Pi^{*}}\right)^{\beta_{1}}\left\langle\left[\frac{(1-\tau) \Pi^{*}}{r+\lambda}-(1-\Omega \tau) I\right]+\right.  \tag{13}\\
& \left.+\left\{\tau-[\tau+(r+\lambda) v]\left(\frac{\Pi^{*}}{\widetilde{\Pi}^{j}}\right)^{\beta_{2}(\lambda)}\right\} \frac{C}{r+\lambda}\right\rangle+ \\
& +\left(\frac{\Pi}{\Pi^{*}}\right)^{\beta_{1}}\left\langle\frac{(1-\tau)}{r+\lambda}-\beta_{2}(\lambda)[\tau+(r+\lambda) v] \Pi^{*^{-1}}\left(\frac{\Pi^{*}}{\widetilde{\Pi}^{j}}\right)^{\beta_{2}(\lambda)} \frac{C}{r+\lambda}\right\rangle=0
\end{align*}
$$

Remember that our firm can choose its capital structure. This means that it can optimally set the value of $C$. Again, differentiating (12) with respect to $C$ gives the following first order condition:

$$
\begin{align*}
\frac{\partial W^{j}}{\partial C}= & \left(\frac{\Pi}{\Pi^{*}}\right)^{\beta_{1}}\left\langle\left\{\tau-[\tau+(r+\lambda) v]\left(\frac{\Pi^{*}}{\widetilde{\Pi}^{j}}\right)^{\beta_{2}(\lambda)}\right\} \frac{1}{r+\lambda}\right\rangle  \tag{14}\\
& +\left(\frac{\Pi}{\Pi^{*}}\right)^{\beta_{1}}\left\langle+\beta_{2}(\lambda)[\tau+(r+\lambda) v]\left(\frac{\Pi^{*}}{\widetilde{\Pi}^{j}}\right)^{\beta_{2}(\lambda)} \frac{C}{r+\lambda} \widetilde{\Pi}^{j-1}\right\rangle=0
\end{align*}
$$

[^6]with $j=s, u$. The f.o.c. (13) and (14) cannot be solved explicitly. This is due to the fact that both $\tau$ and $\lambda_{F}$ affect not only a firm's current payoff, but also its contingent evaluation of future events (i.e., default and the project death). Therefore, the joint analysis of real and financial choices requires a numerical approach. ${ }^{16}$

## 5 A numerical analysis

Let us next focus on a numerical analysis aimed at studying the effects of tax depreciation allowances. Table 1 shows the parameter values.

| $r$ | 0.04 |
| :---: | :---: |
| $\sigma$ | $0.20 ; 0.40$ |
| $\tau$ | $0.20 ; 0.30$ |
| $\lambda$ | $0.03 ; 0.10$ |
| $\Delta \lambda$ | $0.01 ; 0.09$ |
| $\lambda_{f}$ | $\lambda+\Delta \lambda$ |
| $\alpha$ | 0.01 |
| $I$ | 1.00 |
| $v$ | 0.05 |

Table 1: The parameter values.
According to Dixit and Pindyck (1994), for the benchmark case we assume $r=0.04$ and $\sigma=0.20$ : these values are consistent with the empirical evidence (see, e.g., Jorion and Goetzman, 1999, and Dimson et al., 2002). Moreover, we assume that the depreciation rate $\lambda$ is either 0.03 or $0.10 .{ }^{17}$ We also assume that the default cost $v$ is $5 \%$ of the firm's value. ${ }^{18}$ Tax depreciation

[^7]allowances are equal to $\lambda_{f} \equiv \lambda+\Delta \lambda$, where $\Delta \lambda \in[0.01 ; 0.09]$ is the additional benefit due to generous depreciation allowances. Below, we will also carry out some sensitivity analyses regarding parameters $\sigma$ and $\tau$. So, $\sigma$ will also be set equal to 0.4 in order to study the effect of higher uncertainty. Finally, we will run our numerical analysis setting $\tau$ equal to either 0.2 or 0.3 . In doing so, we account for recent tax cuts which have led many countries to apply a statutory tax rate between $20 \%$ and $30 \%$.

Tables 2 and 3 show the effects of an increase in $\Delta \lambda$ on $C^{*}, \Pi^{*}$ and the expected time of default, $E(T)$, i.e., the time lag between investment and default, for $\tau$ equal to 0.2 and 0.3 , respectively. ${ }^{19}$ Under both secured and unsecured debt finance, an increase in $\lambda_{F}$ reduces both $C^{*}$ and $\Pi^{*}$. The decrease in the threshold level $\Pi^{*}$ is not surprising: the more generous the tax depreciation allowance, the earlier an investment is made. Our results show that, due to the lower value of $\Pi^{*}$, it is optimal to reduce the optimal coupon. Otherwise, the default risk would be excessive. ${ }^{20}$ Both Table 2 and 3 show that the expected default time is increasing in $\lambda_{F}$ for $\lambda=0,1$ and $\tau=0.20,0.30$.

It is worth noting that, with $\lambda=0.1$, the expected lifetime of the investment project is 10 periods. As shown in Tables 2 and 3 the expected time of default $E(T)$ is almost always higher than 10 . This means that, with a physiological standard deviation (i.e. $\sigma=0.2$ ), default is a negligible event since the project is quite likely to die before.

[^8]|  | Secured Debt |  | Unsecured Debt |  | E(T) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{\Delta} \boldsymbol{\lambda}$ | $\mathbf{C}^{*}$ | $\boldsymbol{\Pi}^{*}$ | $\mathbf{C}^{*}$ | $\boldsymbol{\Pi}^{*}$ | Secured | Unsecured |
| $\mathbf{0 . 0 1}$ | 0.136 | 0.197 | 0.164 | 0.183 | 9.9 | 11.9 |
| $\mathbf{0 . 0 2}$ | 0.136 | 0.192 | 0.163 | 0.180 | 10.0 | 12.0 |
| $\mathbf{0 . 0 3}$ | 0.135 | 0.188 | 0.162 | 0.176 | 10.1 | 12.2 |
| $\mathbf{0 . 0 4}$ | 0.134 | 0.184 | 0.160 | 0.172 | 10.3 | 12.3 |
| $\mathbf{0 . 0 5}$ | 0.134 | 0.180 | 0.159 | 0.168 | 10.4 | 12.4 |
| $\mathbf{0 . 0 6}$ | 0.133 | 0.176 | 0.158 | 0.164 | 10.6 | 12.5 |
| $\mathbf{0 . 0 7}$ | 0.133 | 0.172 | 0.157 | 0.160 | 10.7 | 12.7 |
| $\mathbf{0 . 0 8}$ | 0.132 | 0.168 | 0.155 | 0.156 | 10.9 | 12.8 |
| $\mathbf{0 . 0 9}$ | 0.132 | 0.163 | 0.154 | 0.153 | 11.0 | 13.0 |

Table 2. $\lambda=0.1 ; \tau=0.2 ; \sigma=0.2 ; \mathrm{I}=1 ; \mathrm{r}=0.04 ; \alpha=0.01 ; v=0.05 ; \lambda_{f}=\lambda+\Delta \lambda$

|  | Secured Debt |  | Unsecured Debt |  | E(T) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{\Delta} \boldsymbol{\lambda}$ | $\mathbf{C}^{*}$ | $\boldsymbol{\Pi}^{*}$ | $\mathbf{C}^{*}$ | $\boldsymbol{\Pi}^{*}$ | Secured | Unsecured |
| $\mathbf{0 . 0 1}$ | 0.140 | 0.178 | 0.160 | 0.159 | 10.8 | 12.8 |
| $\mathbf{0 . 0 2}$ | 0.139 | 0.170 | 0.158 | 0.153 | 11.0 | 13.1 |
| $\mathbf{0 . 0 3}$ | 0.138 | 0.163 | 0.156 | 0.146 | 11.4 | 13.3 |
| $\mathbf{0 . 0 4}$ | 0.137 | 0.155 | 0.154 | 0.139 | 11.7 | 13.7 |
| $\mathbf{0 . 0 5}$ | 0.136 | 0.147 | 0.151 | 0.132 | 12.1 | 14.0 |
| $\mathbf{0 . 0 6}$ | 0.135 | 0.140 | 0.149 | 0.126 | 12.5 | 14.4 |
| $\mathbf{0 . 0 7}$ | 0.134 | 0.132 | 0.147 | 0.119 | 12.9 | 14.8 |
| $\mathbf{0 . 0 8}$ | 0.133 | 0.124 | 0.146 | 0.112 | 13.4 | 15.2 |
| $\mathbf{0 . 0 9}$ | 0.132 | 0.116 | 0.144 | 0.105 | 13.9 | 15.7 |

Table 3. $\lambda=0.1 ; \tau=0.3 ; \sigma=0.2 ; \mathrm{I}=1 ; \mathrm{r}=0.04 ; \alpha=0.01 ; v=0.05 ; \lambda_{f}=\lambda+\Delta \lambda$
Tables 4 and 5 (with $\tau$ equal to 0.2 and 0.3 , respectively) show that similar results are obtained with higher volatility (i.e., $\sigma=0.4$ instead of $\sigma=0.2$ ). This means that, when volatility increases (as happened over the last decade), the default risk still decreases as $\lambda_{F}$ rises. As a consequence, a stimulus pack, characterized by higher tax depreciation allowances, does not lead to higher financial instability.

|  | Secured Debt |  | Unsecured Debt |  | E(T) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{\Delta} \boldsymbol{\lambda}$ | $\mathbf{C}^{*}$ | $\boldsymbol{\Pi}^{*}$ | $\mathbf{C}^{*}$ | $\boldsymbol{\Pi}^{*}$ | Secured | Unsecured |
| $\mathbf{0 . 0 1}$ | 0.156 | 0.300 | 0.182 | 0.269 | 7.0 | 11.2 |
| $\mathbf{0 . 0 2}$ | 0.155 | 0.294 | 0.180 | 0.263 | 7.1 | 11.3 |
| $\mathbf{0 . 0 3}$ | 0.153 | 0.288 | 0.177 | 0.258 | 7.2 | 11.4 |
| $\mathbf{0 . 0 4}$ | 0.152 | 0.282 | 0.174 | 0.253 | 7.3 | 11.6 |
| $\mathbf{0 . 0 5}$ | 0.150 | 0.276 | 0.171 | 0.247 | 7.5 | 11.7 |
| $\mathbf{0 . 0 6}$ | 0.148 | 0.270 | 0.169 | 0.242 | 7.6 | 11.8 |
| $\mathbf{0 . 0 7}$ | 0.147 | 0.264 | 0.166 | 0.237 | 7.7 | 12.0 |
| $\mathbf{0 . 0 8}$ | 0.145 | 0.258 | 0.163 | 0.232 | 7.9 | 12.2 |
| $\mathbf{0 . 0 9}$ | 0.143 | 0.251 | 0.161 | 0.226 | 8.0 | 12.4 |

Table 4. $\lambda=0.1 ; \tau=0.2 ; \sigma=0.4 ; \mathrm{I}=1 ; \mathrm{r}=0.04 ; \alpha=0.01 ; v=0.05 ; \lambda_{f}=\lambda+\Delta \lambda$

|  | Secured Debt |  | Unsecured Debt |  | $\mathbf{E ( T )}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{\Delta} \boldsymbol{\lambda}$ | $\mathbf{C}^{*}$ | $\boldsymbol{\Pi}^{*}$ | $\mathbf{C}^{*}$ | $\boldsymbol{\Pi}^{*}$ | Secured | Unsecured |
| $\mathbf{0 . 0 1}$ | 0.160 | 0.283 | 0.170 | 0.239 | 7.7 | 12.1 |
| $\mathbf{0 . 0 2}$ | 0.157 | 0.272 | 0.166 | 0.230 | 7.9 | 12.4 |
| $\mathbf{0 . 0 3}$ | 0.153 | 0.261 | 0.161 | 0.221 | 8.2 | 12.8 |
| $\mathbf{0 . 0 4}$ | 0.150 | 0.250 | 0.156 | 0.212 | 8.5 | 13.2 |
| $\mathbf{0 . 0 5}$ | 0.147 | 0.239 | 0.151 | 0.203 | 8.9 | 13.6 |
| $\mathbf{0 . 0 6}$ | 0.143 | 0.227 | 0.147 | 0.194 | 9.3 | 14.2 |
| $\mathbf{0 . 0 7}$ | 0.139 | 0.215 | 0.142 | 0.185 | 9.8 | 14.8 |
| $\mathbf{0 . 0 8}$ | 0.136 | 0.203 | 0.137 | 0.175 | 10.4 | 15.6 |
| $\mathbf{0 . 0 9}$ | 0.132 | 0.191 | 0.132 | 0.166 | 11.1 | 16.6 |

Table 5. $\lambda=0.1 ; \tau=0.3 ; \sigma=0.4 ; \mathrm{I}=1 ; \mathrm{r}=0.04 ; \alpha=0.01 ; v=0.05 ; \lambda_{f}=\lambda+\Delta \lambda$
Note that, with a higher value of $\sigma$ (i.e., 0.4 rather than 0.2 ), the expected default time is shorter and default may take place before the expected project death, under secured debt finance (since $E(T)<10$ ). In any case however, $E(T)$ is always increasing in $\lambda_{F}$.

Tables 6 to 9 provide the numerical results with $\lambda=0.03$. In this case, the expected lifetime of an investment project is much longer (i.e., $1 / 0.03=33 . \overline{3}$ ). Since the firm is expected to produce $\Pi$ for longer time, the expected time of default is also longer. As can be seen in Table $6, E(T)$ is around 30 periods, under secured debt finance and is much higher than $33 . \overline{3}$ when debt is unsecured. In these cases, default is therefore a negligible event.

Similar findings are shown in Table 7, i.e., when $\tau=0.30$; in this case, the inequality $C^{*}>\Pi^{*}$ holds under unsecured debt finance. Of course, this result
depends on the symmetric treatment of profits and losses. When $C^{*}>\Pi^{*}$ a firm is making losses and, due to the generous tax depreciation allowances, it is highly subsidized. Though this is an unrealistic case, it is quite important since it highlights the fact that, even if investment were undertaken with an initial loss, tax depreciation allowances would still increase $E(T)$. We can thus say that, even under a system that subsidizes loss-making firms, there is no increase in both default and, above all, systemic risk.

|  | Secured Debt |  | Unsecured Debt |  | E(T) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{\Delta} \boldsymbol{\lambda}$ | $\mathbf{C}^{*}$ | $\boldsymbol{\Pi}^{*}$ | $\mathbf{C}^{*}$ | $\boldsymbol{\Pi}^{*}$ | Secured | Unsecured |
| $\mathbf{0 . 0 1}$ | 0.078 | 0.131 | 0.098 | 0.118 | 26.2 | 36.7 |
| $\mathbf{0 . 0 2}$ | 0.077 | 0.126 | 0.096 | 0.113 | 26.9 | 37.3 |
| $\mathbf{0 . 0 3}$ | 0.076 | 0.121 | 0.094 | 0.109 | 27.6 | 38.0 |
| $\mathbf{0 . 0 4}$ | 0.075 | 0.116 | 0.092 | 0.104 | 28.4 | 38.8 |
| $\mathbf{0 . 0 5}$ | 0.074 | 0.111 | 0.090 | 0.100 | 29.2 | 39.5 |
| $\mathbf{0 . 0 6}$ | 0.072 | 0.106 | 0.088 | 0.095 | 30.1 | 40.4 |
| $\mathbf{0 . 0 7}$ | 0.071 | 0.101 | 0.085 | 0.091 | 31.2 | 41.3 |
| $\mathbf{0 . 0 8}$ | 0.070 | 0.096 | 0.083 | 0.086 | 32.4 | 42.5 |
| $\mathbf{0 . 0 9}$ | 0.069 | 0.090 | 0.081 | 0.081 | 33.7 | 43.7 |

Table 6. $\lambda=0.03 ; \tau=0.2 ; \sigma=0.2 ; \mathrm{I}=1 ; \mathrm{r}=0.04 ; \alpha=0.01 ; v=0.05 ; \lambda_{f}=\lambda+\Delta \lambda$

|  | Secured Debt |  | Unsecured Debt |  | E(T) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{\Delta} \boldsymbol{\lambda}$ | $\mathbf{C}^{*}$ | $\boldsymbol{\Pi}^{*}$ | $\mathbf{C}^{*}$ | $\boldsymbol{\Pi}^{*}$ | Secured | Unsecured |
| $\mathbf{0 . 0 1}$ | 0.081 | 0.128 | 0.095 | 0.108 | 27.8 | 38.4 |
| $\mathbf{0 . 0 2}$ | 0.079 | 0.119 | 0.091 | 0.100 | 29.1 | 39.7 |
| $\mathbf{0 . 0 3}$ | 0.077 | 0.109 | 0.088 | 0.092 | 30.6 | 41.3 |
| $\mathbf{0 . 0 4}$ | 0.074 | 0.100 | 0.084 | 0.085 | 32.5 | 43.2 |
| $\mathbf{0 . 0 5}$ | 0.072 | 0.090 | 0.081 | 0.077 | 34.9 | 45.5 |
| $\mathbf{0 . 0 6}$ | 0.070 | 0.081 | 0.077 | 0.069 | 37.9 | 48.3 |
| $\mathbf{0 . 0 7}$ | 0.068 | 0.071 | 0.074 | 0.061 | 41.8 | 52.1 |
| $\mathbf{0 . 0 8}$ | 0.066 | 0.060 | 0.071 | 0.052 | 47.1 | 57.1 |
| $\mathbf{0 . 0 9}$ | 0.064 | 0.049 | 0.068 | 0.044 | 54.9 | 64.2 |

Table 7. $\lambda=0.03 ; \tau=0.3 ; \sigma=0.2 ; \mathrm{I}=1 ; \mathrm{r}=0.04 ; \alpha=0.01 ; v=0.05 ; \lambda_{f}=\lambda+\Delta \lambda$
In Tables 6 and 7 we have seen that the expected default time is in line with the expected project lifetime. This implies that, at least with a physiological standard deviation of $20 \%$, default is almost a negligible event. Let us next see what happens when $\sigma$ rises from $20 \%$ to $40 \%$. This case
is useful to understand the effects of tax depreciation allowances in a more volatile environment (e.g., during the Great Recession).

|  | Secured Debt |  | Unsecured Debt |  | E(T) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{\Delta} \boldsymbol{\lambda}$ | $\mathbf{C}^{*}$ | $\boldsymbol{\Pi}^{*}$ | $\mathbf{C}^{*}$ | $\boldsymbol{\Pi}^{*}$ | Secured | Unsecured |
| $\mathbf{0 . 0 1}$ | 0.105 | 0.230 | 0.102 | 0.199 | 10.3 | 22.5 |
| $\mathbf{0 . 0 2}$ | 0.103 | 0.221 | 0.098 | 0.192 | 10.6 | 22.9 |
| $\mathbf{0 . 0 3}$ | 0.100 | 0.213 | 0.094 | 0.185 | 10.8 | 23.4 |
| $\mathbf{0 . 0 4}$ | 0.097 | 0.205 | 0.090 | 0.178 | 11.1 | 24.0 |
| $\mathbf{0 . 0 5}$ | 0.084 | 0.196 | 0.086 | 0.172 | 11.5 | 24.7 |
| $\mathbf{0 . 0 6}$ | 0.091 | 0.188 | 0.081 | 0.165 | 11.9 | 25.5 |
| $\mathbf{0 . 0 7}$ | 0.088 | 0.179 | 0.077 | 0.158 | 12.3 | 26.5 |
| $\mathbf{0 . 0 8}$ | 0.085 | 0.171 | 0.072 | 0.151 | 12.8 | 27.8 |
| $\mathbf{0 . 0 9}$ | 0.082 | 0.162 | 0.067 | 0.144 | 13.4 | 29.6 |

Table 8. $\lambda=0.03 ; \tau=0.2 ; \sigma=0.4 ; \mathrm{I}=1 ; \mathrm{r}=0.04 ; \alpha=0.01 ; v=0.05 ; \quad \lambda_{f}=\lambda+\Delta \lambda$

|  | Secured Debt |  | Unsecured Debt |  | E(T) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{\Delta} \boldsymbol{\lambda}$ | $\mathbf{C}^{*}$ | $\boldsymbol{\Pi}^{*}$ | $\mathbf{C}^{*}$ | $\boldsymbol{\Pi}^{*}$ | Secured | Unsecured |
| $\mathbf{0 . 0 1}$ | 0.110 | 0.233 | 0.095 | 0.186 | 10.8 | 23.6 |
| $\mathbf{0 . 0 2}$ | 0.105 | 0.218 | 0.088 | 0.175 | 11.3 | 24.7 |
| $\mathbf{0 . 0 3}$ | 0.100 | 0.203 | 0.081 | 0.165 | 11.9 | 26.2 |
| $\mathbf{0 . 0 4}$ | 0.094 | 0.187 | 0.073 | 0.154 | 12.6 | 28.4 |
| $\mathbf{0 . 0 5}$ | 0.088 | 0.171 | 0.063 | 0.144 | 13.7 | 32.3 |
| $\mathbf{0 . 0 6}$ | 0.081 | 0.155 | 0.053 | 0.134 | 15.2 | 39.3 |
| $\mathbf{0 . 0 7}$ | 0.073 | 0.137 | 0.060 | 0.110 | 17.9 | 35.5 |
| $\mathbf{0 . 0 8}$ | 0.062 | 0.116 | 0.066 | 0.101 | 24.9 | 33.1 |
| $\mathbf{0 . 0 9}$ | 0.068 | 0.109 | 0.073 | 0.097 | 22.1 | 31.3 |

Table 9. $\lambda=0.03 ; \tau=0.3 ; \sigma=0.4 ; \mathrm{I}=1 ; \mathrm{r}=0.04 ; \alpha=0.01 ; v=0.05 ; \lambda_{f}=\lambda+\Delta \lambda$
As shown in Tables 8 and 9, the expected default timing is shorter with $\sigma=0.4$ (compare these results with those contained in Tables 2 and 3). This is not surprising since, with higher volatility, the probability that a firm's EBIT reaches the default threshold level is higher. This means that, when the economic environment is more volatile, the probability of default is higher. However, we can see that an increase in $\lambda_{F}$ still delays default, thereby offsetting the effects of higher aggregate volatility.

As a robustness check, we have finally analyzed the effects of tax depreciation allowances when $C$ is given: this exercise is necessary to see what
happens when firms cannot choose the optimal leverage, for various reasons. To sum up, if $C$ is less than $C^{*},{ }^{21}$ the expected time of default is longer: this is due to the fact that a lower coupon entails a lower default threshold point (either $\widetilde{\Pi}^{u}$ or $\widetilde{\Pi}^{s}$ ). The converse is true when the inequality $C>C^{*} .{ }^{22}$ Our results show that, in the cases examined (with $C \gtrless C^{*}$ ), an increase in $\lambda_{F}$ always causes a rise in $E(T)$.

## 6 Conclusion

In this article, we have studied the effects of tax depreciation allowances on a representative firm that can decide both when to invest and how much to borrow. The firm is aware that, on the one hand, it can benefit from generous depreciation allowances and, on the other hand, it may face default risk.

As expected, accelerated tax depreciation stimulates investment. Its effect on financial decision, however, is less easy to predict. This is due to the fact that tax tools (i.e. $\tau$ and $\lambda_{F}$ ) affect not only a firm's current payoff, but also its contingent evaluation of future events (i.e., default and the project death). To analyze the impact of a stimulus pack on the capital structure we have thus used a numerical approach. As we have shown, generous tax depreciation allowances reduce the propensity to borrow and, in most cases, reduces default risk. In other words, an increase in $\lambda_{F}$ reduces both $C^{*}$ and $\Pi^{*}$. This means that it is optimal to reduce the amount of debt and, at the same time, to invest earlier. The former effect reduces the default risk, while the converse is true for the latter. Our results show that the former effect dominates the latter and hence the expected default time is increasing in $\lambda_{F}$. Similar results are obtained assuming higher volatility (i.e., $\sigma=0.4$ instead of $\sigma=0.2$ ). This means that, even when volatility increases (as happened during the Great Recession), the default risk is still decreasing in $\lambda_{F}$. Therefore, we can state that a stimulus pack, characterized by higher tax depreciation allowances, does not lead to higher financial instability.

[^9]
## A The derivation of (6)

Solving (5) we obtain

$$
E^{j}(\Pi ; C)= \begin{cases}0 & \Pi<\widetilde{\Pi}^{j}  \tag{15}\\ {\left[(1-\tau)\left(\frac{\Pi}{\delta+\lambda}-\frac{C}{r+\lambda}\right)+\tau \Omega I\right]+\sum_{i=1}^{2} A_{i} \Pi^{\beta_{i}(\lambda)}} & \Pi>\widetilde{\Pi}^{j}\end{cases}
$$

where $\Omega \equiv \frac{\lambda_{F}}{r+\lambda}$. Moreover, $\beta_{1}(\lambda)=\frac{1}{2}-\frac{\alpha}{\sigma^{2}}+\sqrt{\left(\frac{\alpha}{\sigma^{2}}-\frac{1}{2}\right)^{2}+\frac{2(r+\lambda)}{\sigma^{2}}}>1$ and $\beta_{2}(\lambda)=\frac{1}{2}-\frac{\alpha}{\sigma^{2}}-\sqrt{\left(\frac{\alpha}{\sigma^{2}}-\frac{1}{2}\right)^{2}+\frac{2(r+\lambda)}{\sigma^{2}}}<0$ are the roots of the characteristic equation $\Psi(\beta) \equiv \frac{1}{2} \sigma^{2} \beta(\beta-1)+\alpha \beta-(r+\lambda)=0$. As can be seen, the beforedefault value of equity consists of two terms: the perpetual rent, in square brackets, and $\sum_{i=1}^{2} A_{i} \Pi^{\beta_{i}(\lambda)}$. Let us next calculate $A_{1}$ and $A_{2}$. In the absence of any financial bubbles, $A_{1}$ is nil (see Dixit and Pindyck, 1994). Therefore, setting $A_{1}=0$ we can rewrite (15) as

$$
E^{j}(\Pi ; C)= \begin{cases}0 & \text { after default }  \tag{16}\\ {\left[(1-\tau)\left(\frac{\Pi}{\delta+\lambda}-\frac{C}{r+\lambda}\right)+\tau \Omega I\right]+A_{2} \Pi^{\beta_{i}(\lambda)}} & \text { before default. }\end{cases}
$$

To calculate $A_{2}$, we must note that when default occurs (i.e., when $\Pi$ drops to $\widetilde{\Pi}^{j}$ ), we have

$$
\begin{equation*}
E\left(\widetilde{\Pi}^{j} ; C\right)=0 \tag{17}
\end{equation*}
$$

with $\widetilde{\Pi}^{j}=\widetilde{\Pi}^{s}, \widetilde{\Pi}^{u}$, since the firm is expropriated by the lender (and equity is nil). Substituting (16) into (17) we find $A$. Rearranging then we obtain (6).

## B The derivation of (9)

Using (6) and differentiating (8) gives the following f.o.c.

$$
\begin{align*}
& \frac{\partial E^{u}(\Pi)}{\partial \widetilde{\Pi}^{u}}=-\frac{(1-\tau)}{\delta+\lambda}\left(\frac{\Pi}{\widetilde{\Pi}^{u}}\right)^{\beta_{2}(\lambda)}  \tag{18}\\
& +\beta_{2}(\lambda)\left[(1-\tau)\left(\frac{\widetilde{\Pi}^{u}}{\delta+\lambda}-\frac{C}{r+\lambda}\right)+\tau \Omega I\right]\left(\frac{\Pi}{\widetilde{\Pi}^{u}}\right)^{\beta_{2}(\lambda)}\left(\widetilde{\Pi}^{u}\right)^{-1}=0 .
\end{align*}
$$

Rearranging (18) gives (9).

C An analysis of term $\left(\frac{\Pi}{\tilde{\Pi}^{j}}\right)^{\beta_{2}(\lambda)}$ with $j=s, u$
Using (7) and (9), it is easy to see that the inequality $\widetilde{\Pi}^{u}<\widetilde{\Pi}^{s}$ holds. Applying the negative exponent $\beta_{2}(\lambda)$ to this inequality gives $\widetilde{\Pi}^{u^{\beta_{2}(\lambda)}}>\widetilde{\Pi}^{\beta_{2}(\lambda)}$. This implies that, for any initial value of $\Pi$, the inequality always holds $\left(\frac{\Pi}{\tilde{\Pi}^{u}}\right)^{\beta_{2}(\lambda)}<\left(\frac{\Pi}{\tilde{\Pi}^{s}}\right)^{\beta_{2}(\lambda)}$. Moreover, differentiating the threshold points with respect to $\lambda_{F}$ gives:

$$
\begin{aligned}
\frac{\partial \widetilde{\Pi}^{s}}{\partial \lambda_{F}} & =-\frac{\tau}{1-\tau} I<0 \\
\frac{\partial \widetilde{\Pi}^{u}}{\partial \lambda_{F}} & =-\frac{\beta_{2}(\lambda)}{\beta_{2}(\lambda)-1} \frac{\delta+\lambda}{r+\lambda} \frac{\tau}{1-\tau} I<0 .
\end{aligned}
$$

Given these results it is easy to see that $\left|\frac{\partial \widetilde{\Pi}^{u}}{\partial \lambda_{F}}\right|<\left|\frac{\widetilde{\Pi}^{s}}{\partial \lambda_{F}}\right|$.

## D The derivation of (10)

Using (4), applying Itô's Lemma and rearranging gives:
$(r+\lambda) D^{j}(\Pi ; C)= \begin{cases}{\left[(1-\tau) \Pi+\tau \lambda_{F} I\right]+\alpha \Pi D_{\Pi}(\Pi ; C)+\frac{\sigma^{2}}{2} \Pi^{2} D_{\Pi \Pi}(\Pi ; C)} & \Pi<\widetilde{\Pi}^{j}, \\ C+\alpha \Pi D_{\Pi}(\Pi ; C)+\frac{\sigma^{2}}{2} \Pi^{2} D_{\Pi \Pi}(\Pi ; C) & \Pi>\widetilde{\Pi}^{j} .\end{cases}$
Solving (19) one obtains:

$$
D^{j}(\Pi ; C)= \begin{cases}\frac{(1-\tau) \Pi}{\delta+\lambda}+\tau \Omega I+\sum_{i=1}^{2} B_{i} \Pi^{\beta_{i}(\lambda)} & \Pi<\widetilde{\Pi}^{j}  \tag{20}\\ \frac{C}{r+\lambda}+\sum_{i=1}^{2} D_{i} \Pi^{\beta_{i}(\lambda)} & \Pi>\widetilde{\Pi}^{j}\end{cases}
$$

where terms $\sum_{i=1}^{2} B_{i} \Pi^{\beta_{i}(\lambda)}$ and $\sum_{i=1}^{2} D_{i} \Pi^{\beta_{i}(\lambda)}$ measure the contingent value of future events after and before default, respectively. To calculate $B_{2}$ we use the boundary condition $D^{j}(0 ; C)=0$, which means that, when $\Pi$ falls to zero, the lender's post-default claim is nil, and so we have $B_{2}=0$. In the
absence of any financial bubble, we also have $B_{1}=D_{1}=0$. To calculate $D_{2}$ we let the pre-default branch of (20) meet with its after-default one, net of the default cost $v C$, at point $\Pi=\widetilde{\Pi}^{j}$, with $j=s$, $u$, i.e.,

$$
\begin{equation*}
\frac{C}{r+\lambda}+D_{2} \widetilde{\Pi}^{j^{\beta_{2}(\lambda)}}=\left[\frac{(1-\tau) \widetilde{\Pi}^{j}}{\delta+\lambda}+\tau \Omega I\right]-v C \tag{21}
\end{equation*}
$$

Solving (21) for $D_{2}$ gives

$$
D_{2}=\left[\frac{(1-\tau) \widetilde{\Pi}^{j}}{\delta+\lambda}+\tau \Omega I-\frac{C}{r+\lambda}-v C\right] \widetilde{\Pi}^{j^{-\beta_{2}(\lambda)}}, \text { with } j=s, u
$$

and substituting this solution into (20) we obtain (10).

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[^0]:    ${ }^{1}$ As stressed by Bhamra et al. (2010), there is a link between firms' capital structure and systemic risk. In particular, they find that leverage accounts for most of the macroeconomic risk relevant for predicting defaults. Another interesting article is Keen et al. (2010), which studies the relationship between taxation and the financial crisis. In particular, the authors stress the fact that tax distortions did not cause the financial crisis. However, they led to higher leverage and more complexity, with some negative drawbacks.
    ${ }^{2}$ See, for example, Mauer and Ott (2000), Miao (2005), Lambrecht (2001), Lambrecht and Perraudin (2003), Mauer and Sarkar (2005), Moretto and Panteghini (2007) and Panteghini (2007a,b).
    ${ }^{3}$ See, for example, Niemann (1999) and Niemann and Sureth (2005).
    ${ }^{4}$ Danielova and Sarkar (2012) study the effects of investment incentives and tax incentives. They show that when debt financing is possible, it is generally optimal to use a combination of tax reduction and investment subsidy. However, they do not investigate tax effects on the default risk and, therefore, disregard systemic risk.

[^1]:    ${ }^{5}$ We leave debt renegotiation for further research.
    ${ }^{6}$ The quality of results does not change if, according to Leland (1994), we assume that the default sunk cost is proportional to a firm's value. In both cases, the value of a distressed firm is proportional to the coupon $C$.

[^2]:    ${ }^{7}$ For simplicity, we do not account for rules aimed at limiting the deductibility of interest payments. Again, this topic is left for further research.
    ${ }^{8}$ Notice that Assumption 8 encourages the investment by loss-making firms. In principle, this tax system would undermine the quality of businesses and increase the probability of default. From our point of view, Assumption 8 describes the most unfavorable scenario, with higher financial instability.

[^3]:    ${ }^{9}$ For further details on mathematical steps see Dixit and Pindyck (1994) and Panteghini (2006, 2007a).
    ${ }^{10}$ Graham and Harvey (2001) show that more than $63 \%$ of the US firms surveyed state that debt maturity is aimed at matching with assets' lifetime. Therefore, our assumption is realistic.
    ${ }^{11}$ For further details on debt covenants see Brennan and Schwartz (1977), Smith and Warner (1979) and Leland (1994).

[^4]:    ${ }^{12}$ As pointed out by Smith and Warner (1979, p. 127) "[s]ecuring debt gives bondholders title to pledged assets until the bonds are paid in full".
    ${ }^{13}$ If shareholders are risk neutral, in equilibrium we have $\delta=r-\alpha>0$. If however they are risk-averse, the dividend yield is $\delta>r-\alpha$ (see McDonald and Siegel, 1984 and 1985).

[^5]:    ${ }^{14}$ See Appendix C.

[^6]:    ${ }^{15}$ Note that $\beta_{1}$ is unaffected by the Poisson process. This is due to the fact that investment timing does not affect the expected lifetime of a firm's project.

[^7]:    ${ }^{16}$ The optimal coupons under both secured and unsecured, namely, $C^{u}$ and $C^{s}$, are such that the inequality $C^{u}>C^{s}$ holds. This is due to the fact that, under unsecured debt financing, a company can decide when to default. Given its higher financial flexibility, therefore, a company can choose a higher leverage ratio (see Panteghini, 2007b).
    ${ }^{17}$ The economic depreciation rate $\lambda$ usually ranges from 0 to 0.10 . This range is in line with the average depreciation rates reported in Fixed Reproducible Tangible Wealth in the United States, 1925-1989 (10\% for equipment and $5 \%$ for structures) and is applied, for example, by Caballero and Engels (1999).
    ${ }^{18}$ It is worth noting that the value of $v$ may depend on several factors that make it range from $5 \%$ to $20 \%$ (see Panteghini, 2007b). For this reason we ran simulations with different values of $v$. However, the quality never changed. For this reason we simply use $v=0.05$ with no loss of generality.

[^8]:    ${ }^{19}$ We calculate this by extending the model described in Dixit and Pindyck (1994) and Dixit (1993).
    ${ }^{20}$ Coeteris paribus, the decrease in $C^{*}$ reduces the probability of default at any time $t$. The converse is true when $\Pi^{*}$ drops: the gap between $\Pi^{*}$ and the default threshold point is reduced and therefore default is more likely. If the former effect (on $C^{*}$ ) dominates the latter (i.e., that on $\Pi^{*}$ ), default is delayed. According to Tables 2 and 3 , this is the more plausible case.

[^9]:    ${ }^{21}$ The inequality $C<C^{*}$ may hold because of liquidity constraints or a troubled access to the credit market (i.e., small businesses may have difficulties in debt financing their activity).
    ${ }^{22}$ The inequality $C>C^{*}$ may hold when there are no credit constraints; however shareholders have no cash to raise equity. Of course, they could invite new shareholders to invest in their representative firm. If however, the old shareholders feared this solution (which might reduce the firm's control), they would prefer a higher leverage.

